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Unsteady Newton-Busemann Flow Theory—Part II: Bodies of Revolution

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Abstract

THE purpose of this paper is to give a complete Newtonian flow theory for unsteady flow past oscillating bodies of revolution of general shape at very high Mach numbers. This is done by adding a centrifugal force correction to the impact pressures. Exact formulas for the unsteady pressure and the stability derivatives are thus obtained in closed form. The centrifugal force correction, which arises from the curved trajectories followed by the fluid particles in unsteady flow, is shown to be very important and must not be neglected. With its inclusion, the present theory is shown to be in excellent agreement with experimental results for relatively sharp cones; however, the theory gives poor agreement with results of experiments in air for bodies having moderate or large-nose bluntness.

Contents

Consider a body of revolution of length ℓ in a uniform hypersonic flow U_∞, which performs a harmonic pitching oscillation with displacement angle of pitch $\theta(t)=\bar{\theta}e^{ikt}$ around zero mean angle of attack about the pivot axis C. Simultaneously, the pivot axis undergoes a harmonic plunging oscillation with flight-path angle $\gamma(t)=\bar{\gamma}e^{ikt}$, so that the body is in a combined pitching and plunging motion. In this paper all lengths are scaled by ℓ, time t by ℓ/U_∞, and pressure p by $\rho_\infty U_\infty^2$. Only small amplitude slow oscillations are considered, and terms of $0(\bar{\theta}^2, \bar{\gamma}^2, \bar{\theta}\bar{\gamma}, \bar{\theta}k^2, \bar{\gamma}k^2)$ are neglected.

The aim is to calculate the resulting unsteady flow based on the Newton-Busemann flow model, which is the exact limiting form of gasdynamic theory in the double limit as the freestream Mach number $M_\infty \to \infty$ and the specific heat ratio $\gamma \to 1$. The surface pressure in this model consists of two parts: the Newtonian impact pressure and a centrifugal force correction owing to the curved trajectories that fluid particles follow along the surface subsequent to their impact.

A body-fixed cylindrical coordinate system (x,r,ϕ) is used, x being along the axis of revolution of the body. The equation of the body surface is given by $r=f(x)$. The Newtonian impact pressure is obtained as

$$p(x,\phi,t)_{\text{impact}} = \mu^2(x)\{f'^2(x)+2f'(x)\cos\phi[\theta(t)+\gamma(t)] + 2f'(x)\cos\phi\dot{\theta}(t)[x-h+f(x)f'(x)]\} \quad (1)$$

where $\mu(x)=[1+f'^2(x)]^{-1/2}$, and h is the distance of the pivot axis C from the nose of the body. The centrifugal correction can be obtained in a way similar to Ref. 1. The total surface pressure is

$$p(x,\phi,t) = P_0(x)+\cos\phi\{[\theta(t)+\gamma(t)]P_1(x) + \dot{\theta}(t)[P_2(x)-hP_1(x)]+\dot{\gamma}(t)P_3(x)\} \quad (2)$$

in which

$$P_0(x) = \mu^2(x)f'^2(x) + \frac{\kappa(x)}{f(x)}\int_0^x \mu(\xi)f(\xi)f'(\xi)\,d\xi \quad (3a)$$

$$P_1(x) = 2\mu^2(x)f'(x) + \frac{2}{3}\frac{\kappa(x)}{f(x)}\int_0^x \mu(\xi)f(\xi)[1-2f'^2(\xi)]\,d\xi \quad (3b)$$

$$P_2(x) = \mu^2(x)[f(x)+2xf'(x)+3f(x)f'^2(x)] + \frac{\kappa(x)}{f(x)}[J_1(x)-J_2(x)-J_3(x)+J_4(x)] \quad (3c)$$

$$P_3(x) = \frac{\mu(x)}{f(x)}J_5(x) + \frac{\kappa(x)}{f(x)}\left\{-J_2(x)+J_4(x) - \int_0^x J_5'(\xi)[f(x)-f(\xi)]\,d\xi - \int_0^x \frac{f(\xi)}{\mu(\xi)}J_5(\xi)S_2(x,\xi)\,d\xi\right\} \quad (3d)$$

where

$$S_n(x,\xi) = \int_\xi^x \frac{d\eta}{\mu(\eta)f^n(\eta)}, \qquad n=0,2$$

$$J_1(x) = \frac{2}{3}\int_0^x \mu(\xi)f(\xi)\{\xi[1-2f'^2(\xi)]+3f(\xi)f'(\xi)\}\,d\xi$$

$$J_2(x) = \int_0^x f(\xi)S_0(x,\xi)\,d\xi$$

$$J_3(x) = \frac{2}{3}\int_0^x f^3(\xi)S_2(x,\xi)\,d\xi$$

$$J_4(x) = \int_0^x \frac{f^2(\xi)f'(\xi)}{\mu(\xi)}S_0(x,\xi)S_2(x,\xi)\,d\xi$$

$$J_5(x) = \int_0^x \frac{f(\xi)f'(\xi)}{\mu(\xi)}\,d\xi \quad (4)$$

and $\kappa(x)=\mu^3(x)f''(x)$ is the curvature of the body surface.

The function $P_1(x)$ in Eq. (3b), which represents the perturbation pressure at small incidence in steady flow, is identical with that given by Hayes and Probstein[2] (when a misprint in Ref. 2 is corrected: the upper limit of the integral (3.8.17) of Ref. 2 should be x_1 instead of 1). The functions $P_2(x)$ and $P_3(x)$ are new, giving the out-of-phase pressure

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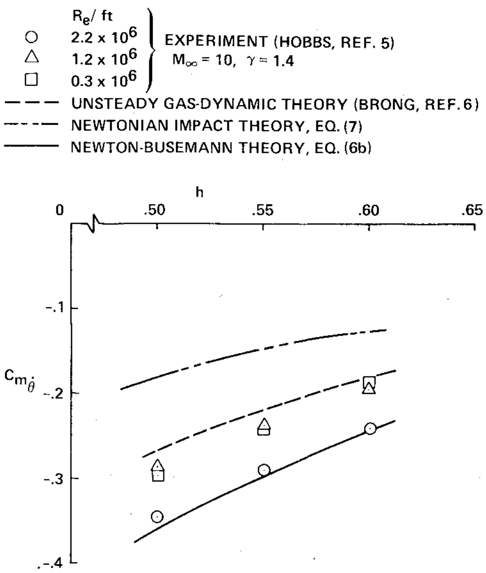

Fig. 1 Comparison of theory and experiment for damping-in-pitch derivative of a 10 deg sharp cone.

components arising from unsteady motions in θ and γ, respectively.

Consider now the stability of the pitching motion of a body of revolution in rectilinear flight $\gamma(t)=0$. The pitching moment is defined as usual $C_m = M/\tfrac{1}{2}\rho_\infty U_\infty^2 S_b \ell$, where S_b is the base area of the body and M is the moment of the surface pressure force about the pivot axis C. The stability derivatives $-C_{m_\theta}$, $-C_{m_{\dot\theta}}$, $-C_{m_q}$, and $-C_{m_{\dot\alpha}}$ are defined as usual[1]; in particular,

$$-C_{m_\theta} = \frac{2}{f^2(1)} \int_0^1 P_1(x) f(x)\,[x-h+f(x)f'(x)]\,\mathrm{d}x \tag{5a}$$

$$-C_{m_{\dot\theta}} = \frac{2}{f^2(1)} \int_0^1 [P_2(x) - hP_1(x)]\,f(x)\,[x-h+f(x)f'(x)]\,\mathrm{d}x \tag{5b}$$

Example: Circular Cones

For a cone of semivertex angle τ, $f(x) = x\tan\tau$, we get

$$-C_{m_\theta} = 2\left(\frac{2}{3} - h\cos^2\tau\right) \tag{6a}$$

$$-C_{m_{\dot\theta}} = \frac{2}{\cos^2\tau}\left[\frac{3}{4} - \frac{5}{3}h\cos^2\tau + (h\cos^2\tau)^2\right] \tag{6b}$$

The stiffness derivative formula, Eq. (6a), has been extensively demonstrated[3] to yield very good agreement with experiment. Equation (6b) for the damping-in-pitch derivative of a cone is new; it generalizes the result of Mahood and Hui[4] to include the dependence on pivot position h.

The importance of the centrifugal force correction for a cone in unsteady flow is clearly demonstrated in Fig. 1, in which the full damping-in-pitch coefficient with centrifugal force correction included [Eq. (6b)] is compared with that resulting from the impact pressure alone; namely, from Eq. (1),

$$\left(-C_{m_{\dot\theta}}\right)_{\text{impact}} = \frac{2}{\cos^2\tau}\left[\frac{1}{2} - \frac{4}{3}h\cos^2\tau + (h\cos^2\tau)^2\right] \tag{7}$$

The difference between the two results is seen to be of the same order of magnitude as the impact pressure contribution. This result casts doubt on the validity of various unsteady Newtonian flow theories that are based on the Newtonian impact pressure alone. The fact that $-C_{m_{\dot\theta}}$ has a significant centrifugal force component, which indicates the influence of flow history, means that even in the Newtonian limit, an accurate estimate of $-C_{m_{\dot\theta}}$ cannot be derived on the basis of a local analysis, such as impact theory, or in fact any theory in which local pressure is said to depend solely on the local flow velocity. This should give pause to those proposing to predict aerodynamic stability derivatives at lower speeds on the basis of similar local analyses.

Also plotted in Fig. 1 are the results of measurements for $-C_{m_{\dot\theta}}$ in air flow at $M_\infty = 10$ by Hobbs[5] and the numerical results by Brong.[6] It is seen that the results based on the complete Newton-Busemann theory are in excellent agreement with the experiments for the highest Reynolds number presented; the numerical results of Brong agree better with the experiments for lower Reynolds numbers. Finally, it can be shown from Eq. (6b) that the pitching motion of a sharp cone of any thickness and about any axis position is dynamically stable in the Newtonian limit.

For blunt cones, the theory indicates that the effect of bluntness is negligibly small for small nose bluntness. Although this is borne out by a comparison with experimental results correlated according to Ericsson's[7] scaling rule (we have used the recent compilation of Khalid and East, Ref. 8), the theory fails to predict the remainder of the important dependence on bluntness indicated by the experimental results. This predictive failure follows from a violation of the principal supposition of the theory, namely, that the bow shock wave conform to the shape of the body.

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